1652

JULY 1972

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PROCUREMENT EXECUTIVE, MINISTRY OF DEFENCE
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UDC 620.193.15 : 532.525.6

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Received for printing 31 May 1972

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AUTHOR'S SUMMARY

The collision of a spherical liquid drop with a plane rigid surface at a right angle is considered and an equation derived for determining the pressure in the zone of contact. The results of experimental determination of the pressure are given which allow the theoretical equation obtained to be used for pressure determination in the solution of practical problems.

CONTENTS

In estimating the wear of surfaces of aircraft due to rain and wear of turbine blades by moist steam we use an equation to determine the maximum pressure arising at the point of impact of a liquid drop against a plane solid surface. Such an equation, proposed in Ref.1, is also given in Refs.2 and 3, i.e.

$$p = \frac{\alpha}{2} \rho V c_{sh} , \tag{1}$$

where α is a coefficient characterising the rigidity of the drop, ρ is the normal density of the liquid, V is the velocity of water particles behind the shock wave taken as equal to the normal collision velocity, c_{sh} is the velocity of propagation of the front of the shock wave in the liquid which increases with the rise in pressure in accordance with the data[4]. However, experimental findings[5,6] show that in the range of collision velocities 150-600 m/s the pressure at the point of impact corresponds to the equation

$$p = \rho V c_{sh} . \tag{2}$$

Thus, experimentally determined pressures at high velocities correspond to a value of 2 for α in equation (1) which is contrary to the physical sense of the coefficient α which cannot be greater than unity[1], and, on the other hand, equation (2) does not agree with experimental findings for collision velocities below 150 m/s.

Let us consider the collision of a fixed spherical drop with a plane surface with an approach velocity V. In Fig.1a the position of the drop and the plane is shown at the moment of impact $(t_0 = 0)$ when the plane is projected on the x-axis of the system of co-ordinates xoz and the lower part of the drop is represented by the arc BOC.

The line 1-1 is the projection of the same plane at the moment t_1. Experiments show that the drop at this moment consists of a spherical undeformed part and a cylindrical part BDEC separated from it by the λ-plane and formed as a result of two simultaneously occurring deformations - the elastic deformation of part of the drop from position BOC to position BAC, and the deformation resulting from the expression of the liquid from position BAC to position BDEC. It is considered that BAC is a parabola, and insofar as the oppositely hatched parts correspond to equal volumes of liquid BD = AF = δ, as shown in Fig.1a.

Following the instant $t_0 = 0$ begins the penetration of the plane into the drop, and this is characterised at the instant t_1 by the segment $AF = \delta$ and by translation of the plane for a distance $\delta' > \delta$. For the time interval $\Delta t = t_1 - t_0$ the relations obtain,

$$\Delta t = \frac{\delta'}{V} = \frac{\delta}{v_1} , \quad \text{whence} \quad v_1 = V \frac{\delta}{\delta'} , \tag{3}$$

where v_1 is the velocity of motion of the plane with respect to the liquid. For the absolute velocity of the liquid v we obtain the expression

$$v = V - v_1 = V - \frac{\delta}{\delta'} V = \frac{\delta' - \delta}{\delta'} V = \alpha V , \tag{4}$$

where

$$\alpha = \frac{\delta' - \delta}{\delta'} < 1 . \tag{5}$$

For velocity v_1

$$v_1 = V - v = (1 - \alpha)V \tag{6}$$

The velocity $\bar{v}$ of motion of the λ-plane in the liquid is twice as high as the velocity v_1 since in time Δt it is translated within the liquid for the distance 2δ

$$\bar{v} = 2(1 - \alpha)V . \tag{7}$$

The mass of liquid drawn into movement with velocity αV is given by

$$m = m_1 + m_2 \tag{8}$$

where $m_1 = \pi R^2 \delta\rho$ is the mass of liquid drawn into motion below the λ-plane (in turn, $R = FD = FE$), and $m_2 = \pi R^2 \rho \left(\frac{z_{max}}{2}\right)$ is the mass of liquid above the λ-plane (Fig.1b) drawn into collision by the shock wave of velocity c_{sh} passing through the drop. Assume that the volume bounded by the parabolic surface, the projection of which on the plane xoz is shown by the dotted line, is equal to the volume of a cylinder of half the height and with the original base. To determine the mass m_2 two states at times t_1 and $t_1 - \Delta' t$ are taken. Correspondingly, $z\lambda$ and z are the co-ordinates of the λ-plane at times

t_1 and $t_1 - \Delta't$; $\Delta'z$ is the distance travelled by the λ-plane in time $\Delta't$; z' is the unknown ordinate of an arbitrary point of the surface bounding the mass of the liquid drawn into motion above the λ-plane; and z'_{max} is the maximum unknown ordinate of this surface at time t_1. Then for the time interval $\Delta't$ the following relations obtain

$$\Delta't = \frac{\Delta'z + z'}{c_{sh}} = \frac{\Delta'z}{\bar{v}},$$

whence

$$z' = \Delta'z\left(\frac{c_{sh}}{\bar{v}} - 1\right). \tag{9}$$

Clearly, $\Delta'z = z_\lambda - z$, where $z_\lambda = \delta' + \delta$ (Fig.1). Taking equation (5) into account, after simple transformations

$$z_\lambda = 2\delta\frac{1 - \frac{\alpha}{2}}{1 - \alpha}.$$

We should note that the expression for z_λ in Ref.1 does not contain the term $\frac{1 - \frac{\alpha}{2}}{1 - \alpha}$ and it is assumed without explanations that $z_\lambda = 2\delta$. This excludes consideration of the elastic deformation of the drop when the ordinate z_λ is being determined. The consequence of this assumption is increasing error in the determination of z_λ as the part played by elastic deformation increases with increasing collision velocity. Furthermore, as the point M at time $t_1 - \Delta't$ is part of a sphere, its ordinate z is determined from the equation of a sphere $r^2 = x^2 + y^2 + (r - z)^2$, and hence when $y = 0$

$$z = r - (r^2 - x^2)^{\frac{1}{2}} \text{ and } \Delta'z = z_\lambda - z = 2\delta\frac{1 - \frac{\alpha}{2}}{1 - \alpha} - r + (r^2 - x^2)^{\frac{1}{2}}.$$

Using equation (9) we can write

$$z' = \left[2\delta\frac{1 - \frac{\alpha}{2}}{1 - \alpha} - r + (r^2 - x^2)^{\frac{1}{2}}\right]\left(\frac{c_{sh}}{v} - 1\right),$$

and then

$$z'_{max} = 2\delta\frac{1 - \frac{\alpha}{2}}{1 - \alpha}\left(\frac{c_{sh}}{v} - 1\right) = 2\delta\frac{1 - \frac{\alpha}{2}}{1 - \alpha}\beta. \tag{10}$$

As a result of substituting the value of z'_{max} in the expression for m_2

$$m_2 = \pi R^2 \beta\delta \frac{1 - \frac{\alpha}{2}}{1 - \alpha} , \tag{11}$$

and after substitution of the values for m_1 and m_2 in the equation for m

$$m = m_1 + m_2 = \pi\rho\delta R^2 + \pi\rho\delta R^2 \beta \frac{1 - \frac{\alpha}{2}}{1 - \alpha} . \tag{12}$$

We now make use of the momentum equation

$$P = \frac{m\Delta v}{\Delta t} , \tag{13}$$

where $\Delta v = v(t_1) - v(t_0) = \alpha V$, since $v(t_0) = 0$.

For the pressure arising in the collision of a drop with a plane we get

$$p = \frac{P}{\pi R^2} = \frac{m\alpha V}{\Delta t \pi R^2} . \tag{14}$$

Taking into account the fact that $\Delta t = \frac{\delta}{(1 - \alpha)V}$, after substitution of the value of m from equation (12) and appropriate transformations

$$p = \frac{\alpha}{2}\left(\frac{1 - \frac{\alpha}{2}}{1 - \alpha} - \alpha\frac{V}{c_{sh}}\right)\rho V c_{sh} = k\rho V c_{sh} . \tag{15}$$

Thus in contradistinction to equation (1), equation (15) contains the term $\left(\frac{1 - \frac{\alpha}{2}}{1 - \alpha} - \alpha\frac{V}{c_{sh}}\right)$ and, for example, even when $V/c_{sh} = 1$, when $\alpha = 0.8$, the ratio $p_{(15)}/p_{(1)} = 2.2$.

To make calculations using equation (15) it is necessary to find values for α or k over a wide range of velocities. Treatment of experiments[7] taking account of an analytical study[8] allows us to conclude that the value of k reaches unity when $\frac{V}{c} \approx 1$ corresponding to the velocity of sound in water $c = 1500$ m/s. The experimental data in Ref.6 confirm this conclusion. One experimental value, $\alpha = 0.4$ when $\frac{V}{c}$ 7.3×10^{-3} is given in Ref.1. The characteristic calculation point corresponding to the intersection of the α and k curves in equation (15), i.e. where $\alpha = k$, corresponds to $\alpha = 0.67$.

We carried out experiments to elucidate further the unknown relationships. At collision velocities up to $V = 6$ m/s drops were hurled at a plane surface of barium titanate piezoceramic crystal of diameter 20 mm and thickness 1.5 mm mounted in a special 'anvil' (Fig.2). The dropper and liquid were located at heights $H_{1,2,3,4} = 1.65$, 1.15, 0.3 and 0.25 metres. The diameter of the drops was 3 mm. Their impact at a constant point of the crystal was assured by a cone mounted above the crystal with an orifice of diameter $d = 4$ mm at the apex. The signal from a transducer was transmitted to a pulse oscillograph mark S-1-19, and a 'Zenith' camera with an attachment was used to take films from the screen. The piezoelectric properties of the barium titanate ceramic subjected to high-voltage polarisation at low mechanical loads (up to 1800 kg/cm^2 at 15°C) are linear[9]. The linearity of the characteristics of the crystal-oscillograph system was confirmed by special experiments. Due to the fact that drops of constant diameter were of different rigidity depending on the velocity of collision the direct proportionality of the height of the blips on the screen to the impact velocity was impaired, and this made it possible, using equation (15) to estimate the change of α in comparison with its known value for the maximum collision velocity and to obtain a number of values of α.

Another method of determining α consisted in hurling drops at the polished surface of a copper disc. The drop in flight passed through the beam of a photoelectric cell which after a time lag to allow the drop to reach the target switched on an SKS-1 camera operating at 4×10^3 frames per second. When the films were processed it was possible to determine the velocity of translation of the plane through the drop when the collision velocity was 5.7 m/s, and from equation (6) to get the value $\alpha = 0.3$. Finally, an SSKS-1 camera operating at 6×10^4 frames per second was used to film the collision of a drop with a target rotated by an electric motor (Fig.2) at $V = 30$ m/s. As a result of processing five films it was possible to determine the velocity of translation of the plane through the drop, α being found to be 0.6. Generalisation of available experimental data and of our own experiments led to our obtaining the dependences of α and k on V (Fig.3) to be used in determining pressures from equation (15), which in contradistinction to equations (1) and (2) is applicable for normal collision velocities from 0 to 600 m/s. At higher velocities, the expenditure of part of the collision energy on deformation of the surface and the drop leads to a certain reduction of pressure in comparison with calculated (theoretical) values. The pressures

calculated from equation (15) for a number of practical cases allow of a satisfactory explanation for the observed intensive erosion damage of surfaces of aircraft components coming into collision with raindrops and of turbine blades eroded by moist steam.

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ILLUSTRATIONS

Fig.1 Diagram for determining the mass of a drop drawn into collision: a - below the λ-plane; b - above the λ-plane.

Fig.2 Apparatus for the study of the collision of a drop with a solid surface: 1 - frame; 2 - cone; 3 - drop catcher; 4 - glass tube; 5 - dropper; 6 - electric motor; 7 - moving target; 8 - cooled target; 9 - piezocrystal target; a - crystal; b - body; c, d - insulators; e - contact disc; f - spring; g - electric terminals; 10 - camera; 11 - light source; 12 - drop. Below right: diagram of method of filming the process of collision of the drop with the target and electronic commutating circuit: I - light source (27 volt lamp); II - photocell type STs8-3; III - cathode follower; IV - amplifier; V - electronic relay; VI - rectifier.

Fig.3 Experimental dependences of the coefficients k and α on V, plotted on the basis of generalised experimental data, 1 - k, 2 - α.

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Fig.1-3

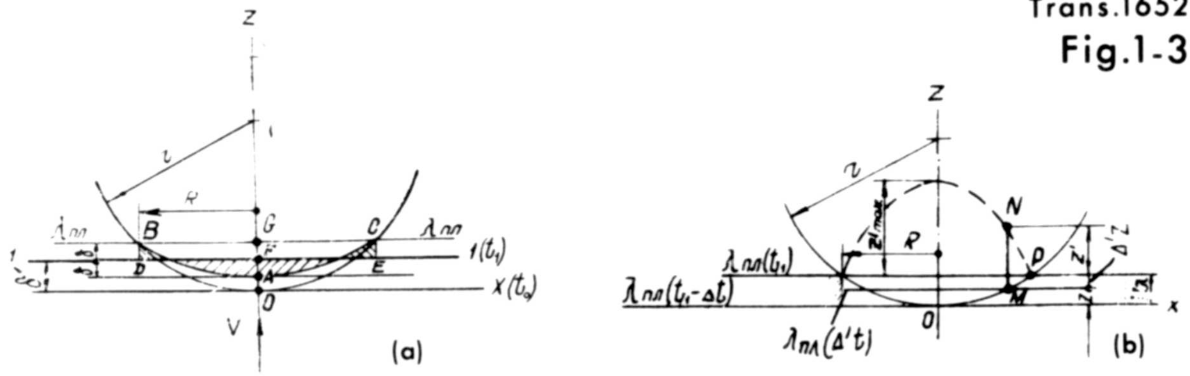

Fig.1

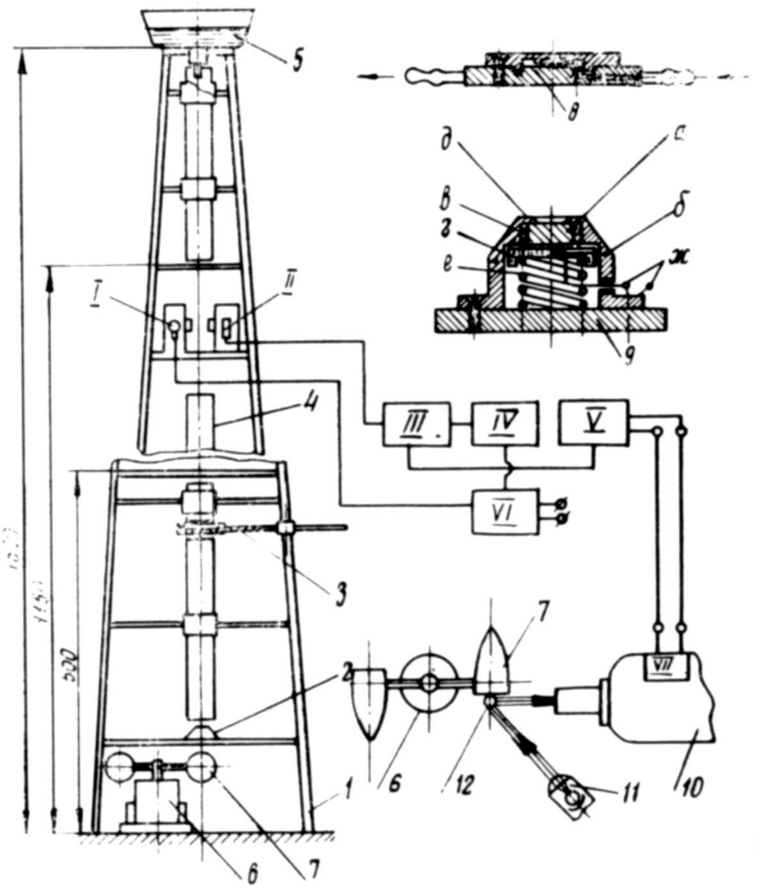

Fig.2

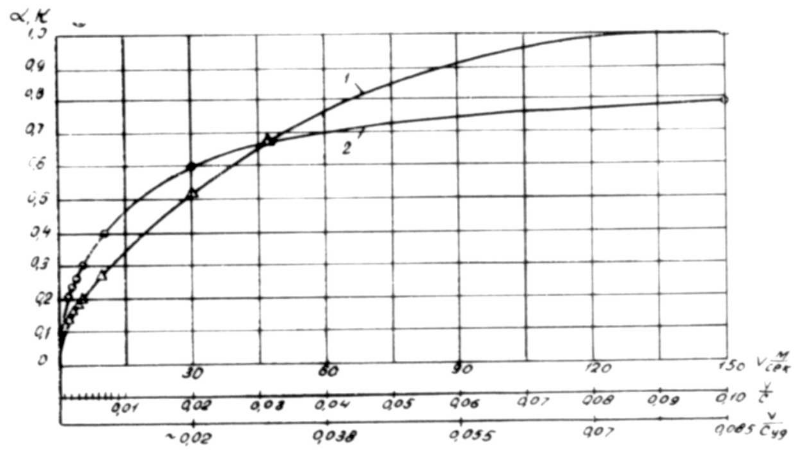

Fig.3